

Introduction to our Tree-System Deductive Apparatus

PHIL 422

Woodbridge

1. The Tree Rules (Phase 1)

We have nine rules for FOL sentences whose main operator(s) are the *sentential* connectives:

Double Negation (DN):

$$\begin{array}{c} \neg \neg \phi \\ | \\ \phi \end{array}$$

Conjunction:	$(\phi \wedge \psi)$	Negated Conjunction:	$\neg(\phi \wedge \psi)$
	$\begin{array}{c} \\ \phi \\ \psi \end{array}$		$\begin{array}{c} \wedge \\ \neg \phi \quad \neg \psi \end{array}$

Disjunction:	$(\phi \vee \psi)$	Negated Disjunction:	$\neg(\phi \vee \psi)$
	$\begin{array}{c} \vee \\ \phi \quad \psi \end{array}$		$\begin{array}{c} \\ \neg \phi \\ \neg \psi \end{array}$

Conditional:	$(\phi \rightarrow \psi)$	Negated Conditional:	$\neg(\phi \rightarrow \psi)$
	$\begin{array}{c} \rightarrow \\ \neg \phi \quad \psi \end{array}$		$\begin{array}{c} \\ \phi \\ \neg \psi \end{array}$

Biconditional:	$(\phi \leftrightarrow \psi)$	Negated Biconditional:	$\neg(\phi \leftrightarrow \psi)$
	$\begin{array}{c} \leftrightarrow \\ \phi \quad \neg \phi \\ \psi \quad \neg \psi \end{array}$		$\begin{array}{c} \leftrightarrow \\ \phi \quad \neg \phi \\ \neg \psi \quad \psi \end{array}$

We *decompose* a set, Σ , of sentences by first listing the sentences in Σ vertically. Call this the *trunk*. Then, for each non-literal (where a literal is either an *atomic* sentence or a *negated* atomic sentence), apply the appropriate rule to decompose it into its simpler parts. When you apply a rule, mark the processed sentence with a “✓”. Keep applying rules until you have only literals.

A branch *closes* if it contains some atomic sentence, ‘ χ ’, and its negation, ‘ $\neg\chi$ ’.

We mark a closed branch with “⊗”.

A branch is complete if it is either closed OR all of its non-literal wffs have been checked.

If a branch is complete and not closed, we call it open (really, we should call it a “completed open branch”). We mark completed open branches with “○”.

A tree is closed if **all** of its branches are closed.

A tree is open if **at least one** of its branches is a completed open branch.

2. Syntactic Assessments

I. Theorems, Anti-theorems and Neutrals

We can test whether a wff has the syntactic status of being a theorem, an anti-theorem, or a neutral by using trees. To do so, place the wff, ' ϕ ', in the trunk (so, $\Sigma = \{\phi\}$). Then, decompose until all branches are completed. Check for closed branches. Next, put the negation of the wff in the trunk of a new tree (so, $\Sigma = \{\neg\phi\}$). Then, decompose until all branches are completed. Finally, check for closed branches.

Definitions:

- A wff, ' ϕ ', is a theorem (written ' $\vdash\phi$ ') iff its tree is open and its negation's tree is closed.
- A wff, ' ϕ ', is an anti-theorem iff its tree is closed and its negation's tree is open.
- A wff, ' ϕ ', is a neutral iff both its tree and its negation's tree are open.

II. Compatible and Incompatible Sets

We can test whether a set of wffs, Σ , is syntactically compatible using trees. To do so, first, place all of the members of Σ in the trunk. Then decompose the trunk until all the branches are complete. Check for closed branches.

Definitions:

- A set of wffs, Σ , is compatible iff its completed tree is open.
- A set of wffs, Σ , is incompatible iff its completed tree is closed.

(As will emerge, compatibility and incompatibility are the basic syntactic relations for our deductive apparatus, instead of derivability, which is defined in terms of incompatibility.)

III. Establishments and Non-establishments

Definitions:

- A set of wffs, Γ , syntactically establishes a wff, ' ϕ ', iff the set of wffs, $\Sigma = \Gamma \cup \{\neg\phi\}$, is incompatible.
- A set of wffs, Γ , syntactically non-establishes a wff, ' ϕ ', iff the set of wffs, $\Sigma = \Gamma \cup \{\neg\phi\}$, is compatible.

Note: We express that Γ establishes ' ϕ ' using the single turnstile, as follows: $\Gamma \vdash \phi$. So, establishment amounts to the derivability relation (when ' ϕ ' is derivable from Γ).

IV. Coupled and Uncoupled Pairs

Definitions:

- A pair of wffs, ' ϕ ' and ' ψ ', are syntactically coupled iff $\phi \vdash \psi$ and $\psi \vdash \phi$ (i.e., $\phi \dashv\vdash \psi$).
- A pair of wffs, ' ϕ ' and ' ψ ', are syntactically uncoupled iff either $\phi \vdash \psi$ fails or $\psi \vdash \phi$ fails.