

PRACTICE PROBLEMS FOR TREES

PHIL 422

I.2 III.1 IV.2

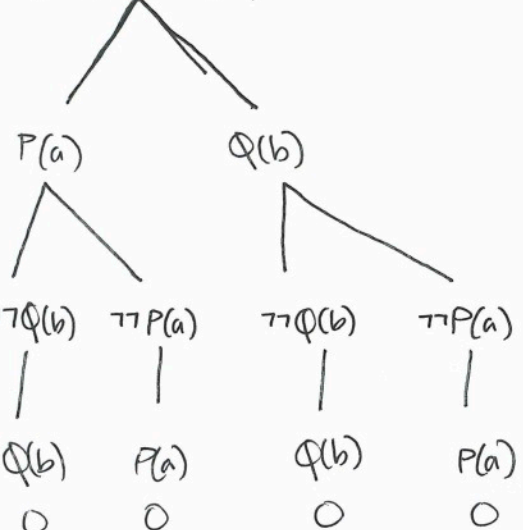
I.2

THEOREM/ANTI THEOREM/NEUTRAL?

$$((P(a) \vee Q(b)) \leftrightarrow \neg(\neg Q(b) \wedge \neg P(a))) \checkmark$$

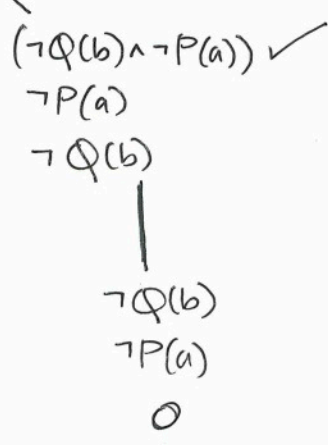
$$(P(a) \vee Q(b)) \checkmark$$

$$\neg(\neg Q(b) \wedge \neg P(a)) \checkmark$$



$$\neg(P(a) \vee Q(b)) \checkmark$$

$$\neg\neg(\neg Q(b) \wedge \neg P(a))$$



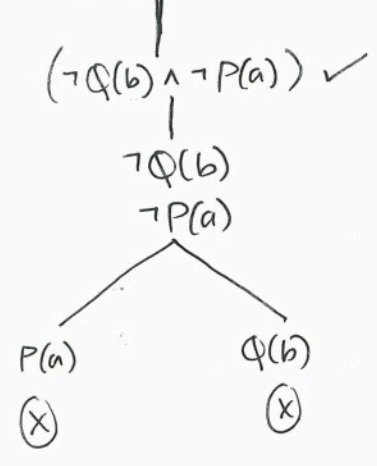
THE WFF IS A THEOREM
BECAUSE ITS TREE IS
OPEN AND ITS
NEGATION'S TREE IS
CLOSED.

TREE IS
OPEN

$$\neg((P(a) \vee Q(b)) \leftrightarrow \neg(\neg Q(b) \wedge \neg P(a))) \checkmark$$

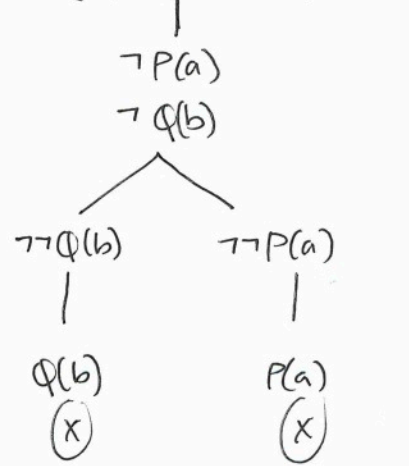
$$(P(a) \vee Q(b)) \checkmark$$

$$\neg\neg(\neg Q(b) \wedge \neg P(a)) \checkmark$$



$$\neg(P(a) \vee Q(b)) \checkmark$$

$$\neg(\neg Q(b) \wedge \neg P(a)) \checkmark$$



TREE IS
CLOSED

III.1

$P(a) \wedge Q(b), (P(a) \rightarrow R(a)), \neg R(a) \vdash S(b)$

$P(a) \wedge Q(b) \checkmark$
 $(P(a) \rightarrow R(a)) \checkmark$

$\neg R(a)$
 $\neg S(b)$

$P(a)$
 $Q(b)$

$R(a)$
 (X)

$\neg P(a)$
 (X)

ESTABLISHMENT/NON-ESTABLISHMENT

$\Gamma = \{ (P(a) \wedge Q(b)), (P(a) \rightarrow R(a)), \neg R(a) \}$
 $\phi = \{ S(b) \}$

$\Gamma \vdash \phi$ because

$\Sigma = \Gamma \cup \{ \neg \phi \}$ is
incompatible [the tree is closed]

IV.2 $(P(a) \rightarrow Q(a)), \neg(P(a) \wedge \neg Q(a))$

$(P(a) \rightarrow Q(a)) \checkmark$
 $\neg \neg(P(a) \wedge \neg Q(a)) \checkmark$

$(P(a) \wedge \neg Q(a)) \checkmark$

$P(a)$
 $\neg Q(a)$

$\neg P(a)$
 (X)

$Q(a)$
 (X)

TREE IS CLOSED,

SO

$(P(a) \rightarrow Q(a)) \vdash \neg(P(a) \wedge \neg Q(a))$

COUPLED/UNCOUPLED

$\phi \vdash \psi$

$\neg(P(a) \wedge \neg Q(a)) \checkmark$

$\neg(P(a) \rightarrow Q(a)) \checkmark$

$P(a)$
 $\neg Q(a)$

$\neg P(a)$
 (X)

$\neg \neg Q(a) \checkmark$

$Q(a)$
 (X)

TREE IS CLOSED,

SO

$\neg(P(a) \wedge \neg Q(a)) \vdash (P(a) \rightarrow Q(a))$

Since $LHS \vdash RHS$ AND $RHS \vdash LHS$, $LHS \dashv\vdash RHS$. Thus, the two sentences are coupled.