

Semantics of the Sentential Connectives

In the following, ' ϕ ' and ' ψ ' are *sentences* (not open wffs) of FOL.

As a refresher, here are the truth-tables, displaying the particular truth-functions that the sentential connectives of FOL express in every interpretation (which is why they are called “logical constants”).

ϕ	ψ	$\neg\phi$	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \rightarrow \psi)$	$(\phi \leftrightarrow \psi)$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Here is an account of the semantics of the connectives in terms of the “truth-conditions” of sentences employing them, relative to an interpretation, I.

- [TC $\neg$] ' $\neg\phi$ ' is true on I iff ' ϕ ' is false on I. ' $\neg\phi$ ' is false on I iff ' ϕ ' is true on I.
- [TC $\wedge$] ' $(\phi \wedge \psi)$ ' is true on I iff ' ϕ ' is true on I and ' ψ ' is true on I. ' $(\phi \wedge \psi)$ ' is false on I iff either ' ϕ ' is false on I OR ' ψ ' is false on I.
- [TC $\vee$] ' $(\phi \vee \psi)$ ' is true on I iff either ' ϕ ' is true on I or ' ψ ' is true on I. ' $(\phi \vee \psi)$ ' is false on I iff BOTH ' ϕ ' is false on I AND ' ψ ' is false on I.
- [TC $\rightarrow$] ' $(\phi \rightarrow \psi)$ ' is true on I iff either ' ϕ ' is false on I or ' ψ ' is true on I. ' $(\phi \rightarrow \psi)$ ' is false on I iff BOTH ' ϕ ' is true on I AND ' ψ ' is false on I.
- [TC $\leftrightarrow$] ' $(\phi \leftrightarrow \psi)$ ' is true on I iff either both ' ϕ ' and ' ψ ' are true on I OR both ' ϕ ' and ' ψ ' are false on I. ' $(\phi \leftrightarrow \psi)$ ' is false on I iff EITHER both ' ϕ ' is false and ' ψ ' is true on I OR both ' ϕ ' is true and ' ψ ' is false on I.