

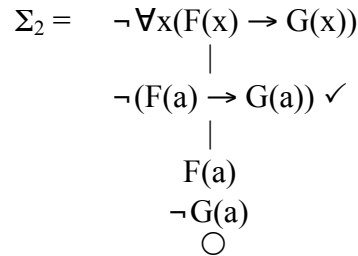
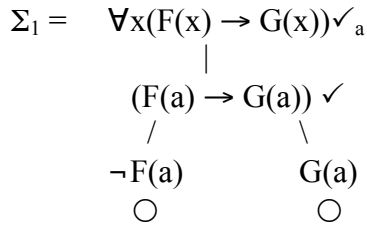
Model Answers for Practice Problems: Trees with Quantifiers and Identity (Phase 2)

PHIL 422

Woodbridge

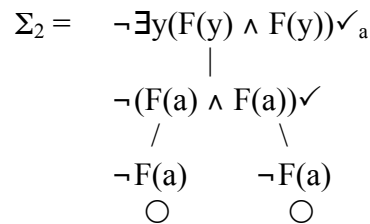
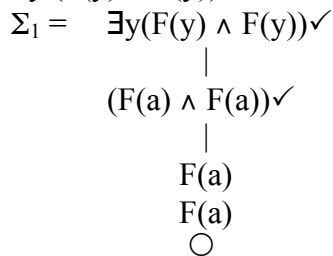
Part I. Theorem, Anti-theorem, or Neutral

1. $\forall x(F(x) \rightarrow G(x))$



Both Trees open: Neutral!

2. $\exists y(F(y) \wedge F(y))$



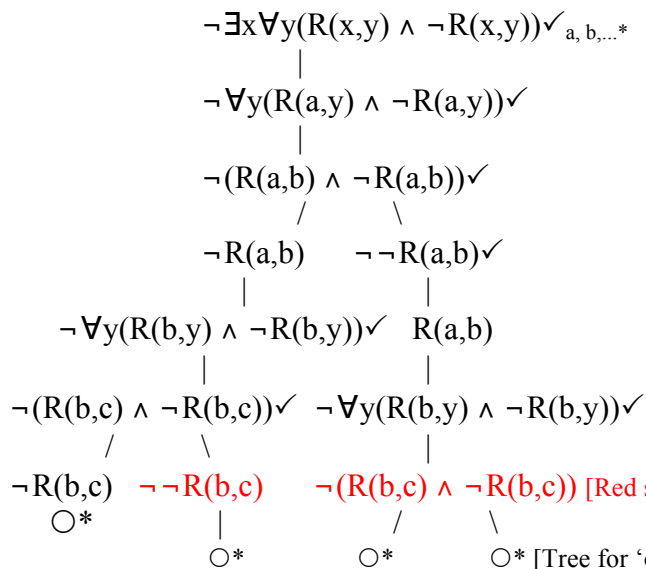
Both Tree open: Neutral!

3. Homework!

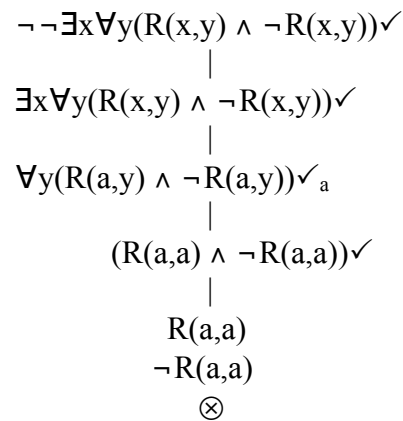
4. Homework!

5. $\neg \exists x \forall y(R(x,y) \wedge \neg R(x,y))$

$$\Sigma_1 = \neg \exists x \forall y(R(x,y) \wedge \neg R(x,y))$$



$$\Sigma_2 = \neg \neg \exists x \forall y(R(x,y) \wedge \neg R(x,y))$$



Tree for ' $\neg \phi$ ' closes: Theorem!

[Red sentences are still unprocessed.]

$\bigcirc^*$ [Tree for ' ϕ ' loops (and branches) infinitely, so it stays **open**.]

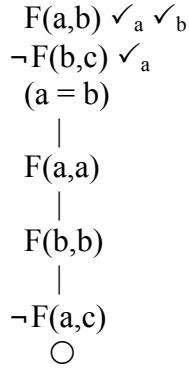
Part II. Establishment or Non-Establishment [#1. for Homework!]

- $$\begin{aligned} & \forall x R(x, x) \checkmark_{a,b} \text{ (complete)} \\ & \forall x \forall y \forall z ((R(x, y) \wedge R(y, z)) \rightarrow R(x, z)) \checkmark_{a,b} \text{ (complete)} \\ & \neg \forall x \forall y (R(x, y) \rightarrow R(y, x)) \checkmark \end{aligned}$$



Part III. Compatible or Incompatible

1. $\{ F(a,b), \neg F(b,c), (a = b) \}$



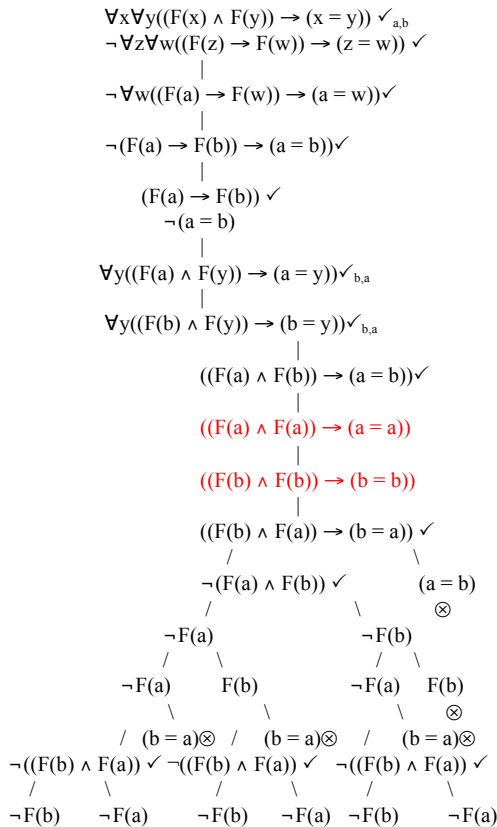
Open Tree, so Compatible!

2. Homework!

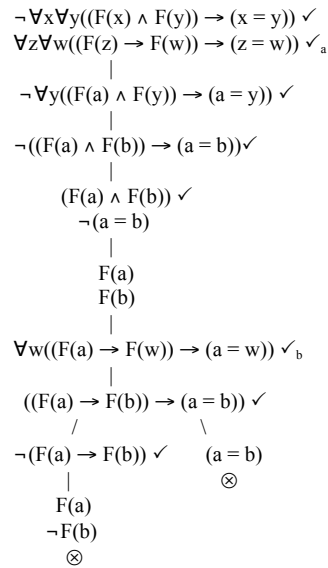
Part IV. Coupled or Uncoupled

1. $\forall x \forall y ((F(x) \wedge F(y)) \rightarrow (x = y)), \forall z \forall w ((F(z) \rightarrow F(w)) \rightarrow (z = w))$
 $\Sigma_1 = \{ \forall x \forall y ((F(x) \wedge F(y)) \rightarrow (x = y)), \neg \forall z \forall w ((F(z) \rightarrow F(w)) \rightarrow (z = w)) \}$
 $\Sigma_2 = \{ \neg \forall x \forall y ((F(x) \wedge F(y)) \rightarrow (x = y)), \forall z \forall w ((F(z) \rightarrow F(w)) \rightarrow (z = w)) \}$

Tree for Σ_1



Tree for Σ_2



[See note on next page.]

Note: The last two unchecked conditionals (in red) on the left will **ultimately** decompose into nodes with “ $\neg F(a)$ ” or “ $\neg F(b)$ ” or “ $(a=a)$ ” or “ $(b=b)$ ” on them. There isn’t space here to add all the branches, but none of those four strings that the processing will end with will conflict with the literals “ $\neg F(a)$ ” or “ $\neg F(b)$ ” on any of the currently open branches. So, this will be an Open Tree. Thus, the left-to-right establishment fails, meaning the sentences are Uncoupled!

2. Homework!