

Practice Problems and Assignment #2
 PHIL 422
 Woodbridge

For Assignment #2, hand in your answers to Ex. I.1, I.3, I.5, I.6; II.1, II.3; III.1, III.2, III.4; IV on Sunday 9/6 by 10pm. Submit your answers as a PDF file uploaded to the Submission link in Canvas (i.e., do NOT send me a link to a Google Drive with your homework file in it).

I. Basic Syntax: For each of the following, demonstrate first whether it is a wff, then say whether it is a *sentence*, of FOL. Explain your answer.

1. $\forall x \exists y H(x,y) \wedge \exists z H(x,z)$
2. $\forall x \forall y \forall z (R(x,y,z) \rightarrow R(x,y,y))$
3. $(P \rightarrow (Q \rightarrow P)) \leftrightarrow \forall x F(x)$
4. $(\exists x (F(x)))$
5. $(\forall x \wedge \forall y) (F(x) \vee F(y))$
6. $\forall x \forall y \forall z (F(x,y) \wedge F(y,z)) \rightarrow F(x,z)$
7. $\forall xyz (Fx \leftrightarrow (Fy \leftrightarrow Gz))$
8. $((P \wedge (Q \wedge (R \wedge S)))$

II. Basic Semantics: Truth Tables. You all learned how to construct truth-tables in PHIL 114 (or an equivalent course). To set up truth-tables in this class, you must always put them into what I will call “standard format”. This first involves putting the “reference columns” for your atomic sentences in alphabetical order from left-to right (e.g., “P”, then “Q”, then “R”). Then you fill in the rows of the right-most of these columns (so, the one under “R”) from the top down, by alternating one “T”, then one “F”, then one “T”, etc. In the next column to the left (so, the one under “Q”) alternate by filling two rows with “T”, then two rows with “F”, etc. In the next column to the left (so, the one under “P”), fill in four rows with “T”, then four rows with “F”. And so on, if there are more reference columns. So, the standard format truth-table for the set of sentences $\{ (P \vee \neg Q), ((Q \leftrightarrow R) \rightarrow P) \}$ (with the final column for each sentence in **RED**) is this.

P	Q	R	$(P \vee \neg Q)$		$((Q \leftrightarrow R) \rightarrow P)$	
T	T	T	T	F	T	T
T	T	F	T	F	F	T
T	F	T	T	T	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	F
F	T	F	F	F	F	T
F	F	T	T	T	F	T
F	F	F	T	T	T	F

Whatever way you learned or were allowed to construct truth-tables before, in this class you must always set them up in standard format. So, with that understood, give standard format truth-tables for the following sets of sentences (one table for each).

1. $\{ (P \wedge Q), (P \rightarrow Q), \neg(Q \vee R) \}$
2. $\{ (P \rightarrow Q), (Q \rightarrow R), (\neg R \rightarrow \neg P) \}$
3. $\{ ((P \rightarrow (Q \rightarrow P)) \vee R) \}$

III. **Concepts:** Answer the following conceptual questions.

1. What does " $\Gamma \vdash \phi$ " mean? (Don't just replace symbols with words; explain the relationship that is said to hold.)
2. What does " $\Gamma \models \phi$ " mean? (Don't just replace symbols with words; explain the relationship that is said to hold.)
3. What is soundness for a formal system? (Give the formal string that states it, but don't *just* do that; explain what it means for a formal system to be sound.)
4. What is completeness for a formal system? (Give the formal string that states it, but don't *just* do that; explain what it means for a formal system to be complete.)

IV. **Mathematical Induction:** Following the pattern of Example 1 provided on the Mathematical Induction Handout (pp. 6-7), identify the elements asked for below from a proof by mathematical induction (MI) of the claim, "Every odd positive integer is such that the sum of it and all the odd positive integers before it, in the ordered sequence of odd positive integers, is equal to the square of the position number of the given odd positive integer in the sequence."

Induction Set:

Partition:

Basis:

(In general, in the Basis you first identify the minimal level of the partition, then specify what case(s) is(are) at that level, and THEN you **prove** that what the claim-to-be-proved says about all members of the induction set holds for (all of) the case(s) at that minimal level.)