

PHIL 422: Advanced Logic

Syntax: Formation Rules for Sentences of First-Order Logic (FOL)

[Note: in the following, double-quotes indicate mention of the symbols enclosed; single-quotes function as permeable corner/Quine quasi-quotes using variables.]

0. (Atomic Sentence/Wff Rule) Any capital letter (with or without subscripts or any number of primes, e.g., “A”, “B”, “A₁”, “A'”, “B'”, “C₃”, “D'''”, “D₅”, “E”, etc.) is an atomic sentence of FOL. Any n-place predicate (capital letter) followed by n names (“a”, “b”, “c”, “d”, “n₁”, “n₂”, “n₃”, ...), inside parentheses, separated by commas, is an atomic sentence of FOL (e.g., “F(a)”, “G(a, b)”, “M(c, d, n₃)”, etc.). **Extension to well-formed formula** (henceforth, **wff**): Any n-place predicate followed by n **individual terms**—either names or **variables** (e.g., “x”, “y”, “z”, “u”, “v”, “w”, “x'”, “y'”, “x₁”, “x₂”, “y₃”, etc.)—inside parentheses, separated by commas, is an atomic **wff** of FOL (e.g., “F(x)”, “G(a, y)”, “M(z'', b, z''')”, etc.). **Special case:** the symbol “=” is the **Identity** predicate. In a wff “=” goes inside the parentheses, flanked by individual terms (e.g., “(a = b)”, “(x = a)”, “(y = z)”).

In the following, “ ϕ ” and “ ψ ” are *metalinguage* variables for wffs of FOL.

1. (Negation Rule) If ‘ ϕ ’ is a sentence/wff of FOL, then so is ‘ $\neg\phi$ ’ [for Teller: ‘ $\sim\phi$ ’] (no parentheses added).
2. (Conjunction Rule) If ‘ ϕ ’ and ‘ ψ ’ are both sentences/wffs of FOL, then so is ‘ $(\phi \wedge \psi)$ ’ [for Teller: ‘ $(\phi \ \& \ \psi)$ ’] (**note:** in 2-5, parentheses are needed).
3. (Disjunction Rule) If ‘ ϕ ’ and ‘ ψ ’ are both sentences/wffs of FOL, then so is ‘ $(\phi \vee \psi)$ ’.
4. (Conditional Rule) If ‘ ϕ ’ and ‘ ψ ’ are both sentences/wffs of FOL, then so is ‘ $(\phi \rightarrow \psi)$ ’ [for Teller: ‘ $(\phi \supset \psi)$ ’].
5. (Biconditional Rule) If ‘ ϕ ’ and ‘ ψ ’ are both sentences/wffs of FOL, then so is ‘ $(\phi \leftrightarrow \psi)$ ’ [for Teller: ‘ $(\phi \equiv \psi)$ ’].
6. (Universal Quantifier Rule) If ‘ ϕ ’ is a wff of FOL and ‘ v ’ is a variable, then ‘ $\forall v\phi$ ’ [for Teller, with parentheses around the quantifier: ‘ $(\forall v)\phi$ ’] is a wff of FOL, and any occurrence of ‘ v ’ in ‘ $\forall v\phi$ ’ is said to be bound.
7. (Existential Quantifier Rule) If ‘ ϕ ’ is a wff of FOL and ‘ v ’ is a variable, then ‘ $\exists v\phi$ ’ [for Teller with parentheses around the quantifier: ‘ $(\exists v)\phi$ ’] is a wff of FOL, and any occurrence of ‘ v ’ in ‘ $\exists v\phi$ ’ is said to be bound.

PC (Parentheses Convention) If after applying and re-applying the Formation Rules to build up to a sentence/wff of FOL, the final rule you apply places parentheses around the whole formula, these outer-most parentheses can be dropped (or kept).