

Handout #1: (Some) Mathematical Preliminaries

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1 Mathematical Preliminaries

We'll assume that everyone at least recognizes the arithmetic operations, even if one can't define them formally. So, you should recognize addition, subtraction, multiplication, and exponentiation.

1.1 Fool's Guide to (some) Naive Set Theory

A set (or class) is a well-defined collection of objects. Use set bracket's to denote a set and its objects: ' $\{o_1, o_2\}$ ' is the set containing the objects ' o_1 ' and ' o_2 '. And we'll often name sets using capital letters or capital Greek letters (with or without subscripts). E.g., ' $A = \{1, 2\}$ ' and ' $B = \{2, 3\}$.' Once we've named a set, we can represent that some object is in the set using the element operation, ' $\in$,' as in ' $1 \in A$ ' and ' $3 \in B$ '. We'll slash through this symbol to show when an object is not in a set, as in ' $3 \notin A$ ' and ' $1 \notin B$.'

Empty Set: The empty set, ' $\emptyset$,' is a set with no member.

Set Equality: Two sets, C and D, are equal just in case they contain all and only the same members.

Example₁: $\{0, 1, 2\} = \{0, 2, 1\} = \{1, 2, 0\} = \{1, 0, 2\} = \{2, 1, 0\} = \dots$

Example₂: $\{\text{Ian}, \text{Kelly}\} = \{\text{Kelly}, \text{Ian}\}$

Extension: We (typically) define sets by their extension (the objects they collect).

Example₃: Let $E = \{1, 5, 9\}$

Example₄: Let $F = \{2, 8, 10\}$

Intension: We (sometimes) define sets/classes by their *intensions*.

Example₅: Let $G = \{x \mid x \text{ is a cat}\}$ [Read: The set of all x such that x is a cat.]

Example₆: Let $H = \{x \mid x \text{ is an even number in } \mathbb{N}\}$

Subset: For two sets, A and B, A is a subset of B if every member of A is a member of B: $A \subseteq B$ iff $\forall \alpha \in A \alpha \in B$.

This is, perhaps, best seen through examples. The following table lists some small sets along with (all) of the subsets.

Set	Subsets
$\{1, 2\}$	$\{1, 2\}, \{1\}, \{2\}, \emptyset$
$\{1, 2, 3\}$	$\{1, 2, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1\}, \{2\}, \{3\}, \emptyset$

You probably noticed the odd fact that every set is a subset of itself. (Also, the empty set is a subset of every set; but that's less odd.)

Subset (continued): Continuing from above.

Example₇: $\{\text{Ian}\} \subseteq \{\text{Ian}, \text{Kelly}\}$

Example₈: $\{\text{Kelly}\} \subseteq \{\text{Ian}, \text{Kelly}\}$

Example₉: $\{\text{Ian}, \text{Kelly}\} \subseteq \{\text{Ian}, \text{Kelly}\}$

Example₁₀: $\emptyset \subseteq \{\text{Ian}, \text{Kelly}\}$

Not Subsets: $A \not\subseteq B =_{\text{def}} \exists \alpha (\alpha \in A \wedge \alpha \notin B)$

Example₁₁: $\{1, 2\} \not\subseteq \{1, 3, 4\}$ [Can you explain why?]

Example₁₂: $\{\text{Miles}, \text{Ian}, \text{Kelly}\} \not\subseteq \{\text{Ian}, \text{Kelly}\}$

Proper Subsets: $A \subset B =_{\text{def}} A \subseteq B$ and $A \neq B$. [Read: A is a proper subset of B if and only if A is a subset of B and A and B aren't equal.]

This is another case where examples illustrate the point better than abstract explanation.

Set	Proper Subsets
$\{1, 2\}$	$\{1\}, \{2\}, \emptyset$
$\{1, 2, 3\}$	$\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1\}, \{2\}, \{3\}, \emptyset$

Proper Subsets (continued): From above.

Example₁₃: $\mathbb{N} \subset \mathbb{Z}$ [The Naturals are a proper subset of the Integers.]

Example₁₄: $\{x | x \text{ is student present in this class}\} \subset \{x | x \text{ is a student at UNLV}\}$

Set Union: $A \cup B =_{\text{def}} \{x | x \in A \vee x \in B\}$

Set Intersection: $A \cap B =_{\text{def}} \{x | x \in A \wedge x \in B\}$

Set Compliment (subtraction): $A - B =_{\text{def}} \{x | x \in A \wedge x \notin B\}$

For examples 15 - 23: Let $I = \{1, 2, 3, 4\}$, $J = \{2, 3, 4, 5\}$ and $K = \{2, 3\}$

Example₁₅: $I \cup J = \{1, 2, 3, 4, 5\}$

Example₁₆: $I \cap J = \{2, 3, 4\}$

Example₁₇: $I - J = \{1\}$

Example₁₈: $I - K = \{1, 4\}$

Example₁₉: $J - K = \{4, 5\}$

Example₂₀: $I \cup K = \{1, 2, 3, 4\} = I$ (because $K \subset I$)

Example₂₁: $I \cap K = \{2, 3\} = K$ (because $K \subset I$)

Example₂₂: $J \cup K = \{2, 3, 4, 5\} = I$ (because $K \subset J$)

Example₂₃: $J \cap K = \{2, 3\} = K$ (because $K \subset J$)

Practice Problems These will be neither collected nor graded.

For these problems, answer true or false. Explain why.

1. $\{1, 2, 3\} \subset \{1, 2, 3\}$
2. $\emptyset \subseteq \{1, 2, 3\}$
3. $\{\emptyset\} \subset \{1, 2, 3\}$

For these problems: Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{3, 4, 5\}$, $D = \{3\}$

1. What is $A \cap B$?
2. What is $A \cup B$?
3. What is $A - D$?
4. What is $(A \cap B) \cup C$?
5. What is $A \cap C$?
6. What is $D - A$?
7. What is $B \cup C$?
8. What is $B - C$?
9. What is $((A \cap B) \cap C) - D$?

1.2 Coming Soon (on another handout): Mathematical Induction

Another mathematical technique we'll use in this class is called mathematical induction. It's really a collection of techniques that allow you to prove things about well-behaved collections, including infinitely large, but well-behaved collections.