

Additional Practice Problems: Semantic Assessments—SOLUTIONS
PHIL 422
Woodbridge

Use the techniques we have employed in doing semantic proofs to answer the following questions. [Several of these are longer than what I'll give you on Monday's test, but they illustrate important points.]

1. What is the semantic status of " $(F(a) \wedge \forall x(F(x) \rightarrow G(x))) \wedge \neg G(a)$ "? Is it L-true, L-false, or contingent?

Part I: Can it be true? If so, then some I makes

1. " $(F(a) \wedge \forall x(F(x) \rightarrow G(x)))$ " true AND
2. " $\neg G(a)$ " true. [TC $\wedge$]

Also, "a" designates $\alpha \in D$ on I.

3. From 2., I makes " $G(a)$ " false. [TC $\neg$]
4. From 1., I makes " $F(a)$ " true AND
5. " $\forall x(F(x) \rightarrow G(x))$ " true. [TC $\wedge$]
6. From 3., $\alpha \notin "G"$ on I. [TC Atomic]
7. From 4., $\alpha \in "F"$ on I. [TC Atomic]
8. From 5., every $\beta \in D$ satisfies " $(F(x) \rightarrow G(x))$ " on I. [TC $\forall$]
9. From 8., for every $\beta \in D$, if ' m ' designates β on I, then ' $(F(m) \rightarrow G(m))$ ' is true on I. [Def. Sat.]
10. From 9., for every $\beta \in D$, if ' m ' designates β on I, then I makes ' $F(m)$ ' false OR makes ' $G(m)$ ' true. [TC $\rightarrow$]
11. From 10., for every $\beta \in D$, $\beta \notin "F"$ on I OR $\beta \in "G"$ on I. [TC Atomic]

Since β is unrestricted, let β be α . Then, by 11., $\alpha \notin "F"$ on I OR $\alpha \in "G"$ on I. [Substitution]

The first option contradicts 7. and the second option contradicts 6. So, I is impossible. So, no I makes the sentence true. Thus, the sentence is L-false. [No need for Part II, asking about falsity.]

2. What is the semantic status of " $\neg \forall y(\neg G(y) \vee \neg(\neg H(y) \vee (\neg F(y) \vee G(y))))$ "? Is it L-true, L-false, or contingent?

Part I: Can it be true? If so, then some I makes

1. " $\forall y(\neg G(y) \vee \neg(\neg H(y) \vee (\neg F(y) \vee G(y))))$ " false. [TC $\neg$]
2. From 1., some $\alpha \in D$ does not satisfy " $(\neg G(y) \vee \neg(\neg H(y) \vee (\neg F(y) \vee G(y))))$ " on I. [TC $\forall$]
3. From 2., for some $\alpha \in D$, if ' m ' designates α on I, then ' $(\neg G(m) \vee \neg(\neg H(m) \vee (\neg F(m) \vee G(m))))$ ' is false on I. [Def. Sat.]
4. From 3., for some $\alpha \in D$, if ' m ' designates α on I, then I makes ' $\neg G(m)$ ' false AND makes ' $\neg(\neg H(m) \vee (\neg F(m) \vee G(m)))$ ' false. [TC $\vee$]
5. From 4., for some $\alpha \in D$, if ' m ' designates α on I, then I makes ' $G(m)$ ' true AND makes ' $(\neg H(m) \vee (\neg F(m) \vee G(m)))$ ' true. [TC $\neg$]
6. From 5., for some $\alpha \in D$, if ' m ' designates α on I, then $\alpha \in "G"$ on I [TC Atomic] AND either I makes ' $\neg H(m)$ ' true OR makes ' $(\neg F(m) \vee G(m))$ ' true. [TC $\vee$]

7. From 6., for some $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then $\alpha \in \text{"G"}$ on I AND either I makes ' $\mathbf{H(m)}$ ' false [TC $\neg$] OR either I makes ' $\neg \mathbf{F(m)}$ ' true OR makes ' $\mathbf{G(m)}$ ' true. [TC $\vee$]
8. From 7., for some $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then $\alpha \in \text{"G"}$ on I AND either $\alpha \notin \text{"H"}$ on I [TC Atomic] OR either I makes ' $\mathbf{F(m)}$ ' false [TC $\neg$] OR $\alpha \in \text{"G"}$ on I. [TC Atomic]
9. From 8., for some $\alpha \in D$, $\alpha \in \text{"G"}$ on I AND [either $\alpha \notin \text{"H"}$ on I OR either $\alpha \notin \text{"F"}$ on I OR $\alpha \in \text{"G"}$ on I].

So, let I set $D = \{1\}$, " $\mathbf{F}$ ": $\emptyset$, " $\mathbf{G}$ ": $\{1\}$, " $\mathbf{H}$ ": $\emptyset$. [Many other interpretations that satisfy the "ORs" differently will also work here.] This makes the sentence true. So it's not L-false.

Part II: Can it be false? If so, then some I makes

1. " $\forall y(\neg \mathbf{G(y)} \vee \neg(\neg \mathbf{H(y)} \vee (\neg \mathbf{F(y)} \vee \mathbf{G(y)})))$ " true. [TC $\neg$]
2. From 1., every $\alpha \in D$ satisfies " $(\neg \mathbf{G(y)} \vee \neg(\neg \mathbf{H(y)} \vee (\neg \mathbf{F(y)} \vee \mathbf{G(y)})))$ " on I. [TC $\forall$]
3. From 2., for every $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then ' $(\neg \mathbf{G(m)} \vee \neg(\neg \mathbf{H(m)} \vee (\neg \mathbf{F(m)} \vee \mathbf{G(m)})))$ ' is true on I. [Def. Sat.]
4. From 3., for every $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then either I makes ' $\neg \mathbf{G(m)}$ ' true OR makes ' $\neg(\neg \mathbf{H(m)} \vee (\neg \mathbf{F(m)} \vee \mathbf{G(m)}))$ ' true. [TC $\vee$]
5. From 4., for every $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then either I makes ' $\mathbf{G(m)}$ ' false OR makes ' $(\neg \mathbf{H(m)} \vee (\neg \mathbf{F(m)} \vee \mathbf{G(m)}))$ ' false. [TC $\neg$]
6. From 5., for every $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then either $\alpha \notin \text{"G"}$ on I [TC Atomic] OR I makes ' $\neg \mathbf{H(m)}$ ' false AND makes ' $(\neg \mathbf{F(m)} \vee \mathbf{G(m)})$ ' false. [TC $\vee$]
7. From 6., for every $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then either $\alpha \notin \text{"G"}$ on I OR I makes ' $\mathbf{H(m)}$ ' true [TC $\neg$] AND makes ' $\neg \mathbf{F(m)}$ ' false AND makes ' $\mathbf{G(m)}$ ' false. [TC $\vee$]
8. From 7., for some $\alpha \in D$, if ' $\mathbf{m}$ ' designates α on I, then $\alpha \notin \text{"G"}$ on I OR [$\alpha \notin \text{"H"}$ on I [TC Atomic] AND I makes ' $\mathbf{F(m)}$ ' true [TC $\neg$] AND $\alpha \notin \text{"G"}$ on I. [TC Atomic]]
9. From 8., for some $\alpha \in D$, $\alpha \notin \text{"G"}$ on I OR [$\alpha \in \text{"H"}$ on I AND $\alpha \in \text{"F"}$ on I AND $\alpha \notin \text{"G"}$ on I]. [Note: nothing is preventing both "OR" options in 9. holding together.]

So, let I be $D = \{1\}$, " $\mathbf{F}$ ": $\{1\}$, " $\mathbf{G}$ ": $\emptyset$, " $\mathbf{H}$ ": $\{1\}$. [Again, other interpretations satisfying the "ORs" differently will also work here.] I makes the sentence false. So it's not L-true.

Since the sentence is neither L-true nor L-false, it is contingent.

3. Is the following sequent correct, i.e., do we have entailment or entailment?

$\{ \forall x(\mathbf{F(x)} \rightarrow \mathbf{G(x)}), \forall y(\mathbf{G(y)} \rightarrow \mathbf{H(y)}), \mathbf{F(a)} \} \models (\mathbf{H(a)} \vee \mathbf{H(b)})$

Check for entailment, that is see if some I makes

1. " $\forall x(\mathbf{F(x)} \rightarrow \mathbf{G(x)})$ " true
2. " $\forall y(\mathbf{G(y)} \rightarrow \mathbf{H(y)})$ " true
3. " $\mathbf{F(a)}$ " true
4. " $(\mathbf{H(a)} \vee \mathbf{H(b)})$ " false.

Also, " $\mathbf{a}$ " designates $\alpha \in D$ on I, and " $\mathbf{b}$ " designates $\beta \in D$ on I. [Note that α and β could be the same object, as far as this condition by itself is concerned, but by using different object variables we also allow that they could be distinct.]

5. From 1., every $\gamma \in D$ satisfies " $(F(x) \rightarrow G(x))$ " on I. [TC $\forall$]
6. From 5., for every $\gamma \in D$, if ' $\mathfrak{m}$ ' designates γ on I, then ' $(F(\mathfrak{m}) \rightarrow G(\mathfrak{m}))$ ' is true on I. [Def. Sat.]
7. From 6., for every $\gamma \in D$, if ' $\mathfrak{m}$ ' designates γ on I, then ' $F(\mathfrak{m})$ ' is false on I OR ' $G(\mathfrak{m})$ ' is true on I. [TC $\rightarrow$]
8. From 7., for every $\gamma \in D$, $\gamma \notin "F"$ on I OR $\gamma \in "G"$ on I. [TC Atomic].
9. From 2., every $\gamma \in D$ satisfies " $(G(y) \rightarrow H(y))$ " on I. [TC $\forall$]
10. From 9., for every $\gamma \in D$, if ' $\mathfrak{n}$ ' designates γ on I, then ' $(G(\mathfrak{n}) \rightarrow H(\mathfrak{n}))$ ' is true on I. [Def. Sat.]
11. From 10., for every $\gamma \in D$, if ' $\mathfrak{n}$ ' designates γ on I, then ' $G(\mathfrak{n})$ ' is false on I OR ' $H(\mathfrak{n})$ ' is true on I. [TC $\rightarrow$]
12. From 11., for every $\gamma \in D$, $\gamma \notin "G"$ on I OR $\gamma \in "H"$ on I. [TC Atomic].
13. From 3., $\alpha \in "F"$ on I. [TC Atomic]
14. From 4., I makes " $H(a)$ " false AND makes " $H(b)$ " false. [TC $\vee$]
15. From 14., $\alpha \notin "H"$ on I AND $\beta \notin "H"$ on I. [TC Atomic]

Since γ is unrestricted, let γ be α . Then, by 8., $\alpha \notin "F"$ on I OR $\alpha \in "G"$ on I. [Substitution]
The first option contradicts 9., so $\alpha \in "G"$ on I [by disjunctive syllogism]. But, by 12., we have $\alpha \notin "G"$ on I OR $\alpha \in "H"$ on I. [Substitution] The first option contradicts our result of $\alpha \in "G"$ on I, leaving $\alpha \in "H"$ on I. But this contradicts 15. So I is impossible, meaning that no entailment I exists. So the entailment holds.

4. Is the following sequent correct, i.e., do we have entailment or entailment?

$$\{ \neg \exists x(F(x) \wedge \neg G(x)), \forall y(H(y) \vee \neg G(y)) \} \models \neg \forall z(F(y) \rightarrow H(z))$$

Check for entailment, i.e., look for some I that makes

1. " $\neg \exists x(F(x) \wedge \neg G(x))$ " true
2. " $\forall y(H(y) \vee \neg G(y))$ " true
3. " $\neg \forall z(F(y) \rightarrow H(z))$ " false.
4. From 1., I makes " $\exists x(F(x) \wedge \neg G(x))$ " false. [TC $\neg$]
5. From 3., I makes " $\forall z(F(y) \rightarrow H(z))$ " true. [TC $\neg$]
6. From 4., every $\alpha \in D$ does not satisfy " $(F(x) \wedge \neg G(x))$ " on I. [TC $\exists$]
7. From 6., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I, then ' $(F(\mathfrak{m}) \wedge \neg G(\mathfrak{m}))$ ' is false on I. [Def. Sat.]
8. From 7., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I, then ' $F(\mathfrak{m})$ ' is false on I OR ' $\neg G(\mathfrak{m})$ ' is false on I. [TC $\wedge$]
9. From 8., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I, then $\alpha \notin "F"$ on I [TC Atomic] OR ' $G(\mathfrak{m})$ ' is true on I. [TC $\neg$]
10. From 9., for every $\alpha \in D$, $\alpha \notin "F"$ on I OR $\alpha \in "G"$ on I. [TC Atomic]
11. From 5., every $\alpha \in D$ satisfies " $(F(y) \rightarrow H(z))$ " on I. [TC $\forall$]
12. From 11., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I, then ' $(F(\mathfrak{m}) \rightarrow H(\mathfrak{m}))$ ' is true on I. [Def. Sat.]
13. From 12., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I, then ' $F(\mathfrak{m})$ ' is false on I OR ' $H(\mathfrak{m})$ ' is true on I. [TC $\rightarrow$]
14. From 13., for every $\alpha \in D$, $\alpha \notin "F"$ on I OR $\alpha \in "H"$ on I. [TC Atomic]

15. From 2., every $\alpha \in D$ satisfies " $(H(y) \vee \neg G(y))$ " on I. [TC $\forall$]
 16. From 15., for every $\alpha \in D$, if ' m ' designates α on I, then ' $(H(m) \vee \neg G(m))$ ' is true on I. [Def. Sat.]
 17. From 16., for every $\alpha \in D$, if ' m ' designates α on I, then ' $H(m)$ ' is true on I OR ' $\neg G(m)$ ' is true on I. [TC $\vee$]
 18. From 17., for every $\alpha \in D$, if ' m ' designates α on I, then $\alpha \in "H"$ on I [TC Atomic] OR ' $G(m)$ ' is false on I. [TC $\neg$]
 19. From 18., for every $\alpha \in D$, $\alpha \in "H"$ on I OR $\alpha \notin "G"$ on I. [TC Atomic]
- From 10., 14., and 19., we can have I as follows: $D = \{1\}$, " F ": $\emptyset$, " G ": $\emptyset$, " H ": $\{1\}$. This I makes all of the sentences in the set on the lhs true, while making the sentence on the rhs false. So this is a case of entailment.

5. Is the following set consistent or inconsistent?

$\{ R(a,b), R(b,a), \forall x \forall y (\neg R(x,y) \rightarrow R(x,x)), \exists x \exists y \neg R(x,y) \}$

Check to see if it is consistent, that is, whether some I makes:

1. " $R(a,b)$ " true
2. " $R(b,a)$ " true
3. " $\forall x \forall y (\neg R(x,y) \rightarrow R(x,x))$ " true
4. " $\exists x \exists y \neg R(x,y)$ " true.

Also, " a " designates $\alpha \in D$ on I, and " b " designates $\beta \in D$ on I. [Again, α and β could be the same object, or they could be distinct objects.]

5. From 1., $\langle \alpha, \beta \rangle \in "R"$ on I. [TC Atomic]
6. From 2., $\langle \beta, \alpha \rangle \in "R"$ on I. [TC Atomic]
7. From 3., every $\gamma \in D$ satisfies " $\forall y (\neg R(x,y) \rightarrow R(x,x))$ " on I. [TC $\forall$]
8. From 7., for every $\gamma \in D$, if ' m ' designates γ on I, then ' $\forall y (\neg R(m,y) \rightarrow (R(m,m)))$ ' is true on I. [Def. Sat.]
9. From 8., for every $\gamma \in D$, if ' m ' designates γ on I, every $\delta \in D$ satisfies ' $(\neg R(m,y) \rightarrow R(m,m))$ ' on I. [TC $\forall$]
10. From 9., for every $\gamma \in D$, every $\delta \in D$ is such that, if ' m ' designates γ on I and ' n ' designates δ on I, then ' $(\neg R(m,n) \rightarrow R(m,m))$ ' is true on I. [Def. Sat.]
11. From 10., for every $\gamma \in D$, every $\delta \in D$ is such that, if ' m ' designates γ on I and ' n ' designates δ on I, then either ' $\neg R(m,n)$ ' is false on I OR ' $R(m,m)$ ' is true on I. [TC $\rightarrow$]
12. From 11., for every $\gamma \in D$, every $\delta \in D$ is such that, if ' m ' designates γ on I and ' n ' designates δ on I, then either ' $R(m,n)$ ' is true on I [TC $\neg$] OR $\langle \gamma, \gamma \rangle \in "R"$ on I. [TC Atomic]
13. From 12., for every $\gamma \in D$, every $\delta \in D$ is such that, either $\langle \gamma, \delta \rangle \in "R"$ on I [TC Atomic] OR $\langle \gamma, \gamma \rangle \in "R"$ on I.
14. From 4., some $\eta \in D$ satisfies " $\exists y \neg R(x,y)$ " on I. [TC $\exists$]
15. From 14., for some $\eta \in D$, if ' m ' designates η on I, then ' $\exists y \neg R(m,y)$ ' is true on I. [Def. Sat.]
16. From 15., for some $\eta \in D$, if ' m ' designates η on I, some $\pi \in D$ satisfies ' $\neg R(m,y)$ ' on I. [TC $\exists$]

17. From 16., for some $\eta \in D$, some $\pi \in D$ is such that, if ' $\mathfrak{m}$ ' designates η on I and ' $\mathfrak{n}$ ' designates π on I, then ' $\neg R(\mathfrak{m}, \mathfrak{n})$ ' is true on I. [Def. Sat.]

18. From 17., for some $\eta \in D$, some $\pi \in D$ is such that, if ' $\mathfrak{m}$ ' designates η on I and ' $\mathfrak{n}$ ' designates π on I, then ' $R(\mathfrak{m}, \mathfrak{n})$ ' is false on I. [TC $\neg$]

19. From 18., for some $\eta \in D$, some $\pi \in D$ is such that $\langle \eta, \pi \rangle \notin "R"$ on I. [TC Atomic]

Since γ and δ from 13. are both unrestricted, consider when each is α and when each is β . [Then we see that the conditions on I from 13. fit with those from 5. and 6.; 13. just tells us to include more in the extension of "R" than 5. and 6. already do.] Since 19. requires that some ordered pair be excluded from the extension of "R" (while 5. and 6. require including pairs), we can see that we again have to have more than one object in the domain, D. But it turns out that we also have to have more than two objects in D. [If D has only two objects, we can satisfy 5., 6., and 13., only by putting all four ordered pairs that D generates in the extension of "R". But then none will be excluded, so the conditions in 19. will not be met.] If the domain has three objects, then we can satisfy 5. and 6. with two of them and meet the second option of 13. with all three objects, while still leaving at least one ordered pair out of the extension in order to satisfy 19. So, have I as follows: $D = \{1, 2, 3\}$, "a" designates 1, "b" designates 2, "R": $\{\langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle\}$. [The first two pairs being members meets the conditions in 5. and 6. The last three pairs being members meets the second option of the conditions in 13. Moreover, because this I leaves the pairs $\langle 1, 3 \rangle$, $\langle 2, 3 \rangle$, $\langle 3, 1 \rangle$, and $\langle 3, 2 \rangle$ out of the extension of "R", it meets (multiple times) the exclusion condition of 19.] This makes all three sentences true, so the set is consistent.

6. Is $\neg \forall x \exists y \neg G(x, y) \models \forall x \exists y G(x, y)$? (That is, are the two sentences equivalent or inequivalent?) Check for inequivalence, that is, check whether there is some I_1 that makes the first true and the second false, or some I_2 that makes the first false and the second true.

Part I: I_1 makes

1. " $\neg \forall x \exists y \neg G(x, y)$ " true AND
2. " $\forall x \exists y G(x, y)$ " false.
3. From 1., I_1 makes " $\forall x \exists y \neg G(x, y)$ " false. [TC $\neg$]
4. From 3., some $\alpha \in D$ does not satisfy " $\exists y \neg G(x, y)$ " on I_1 . [TC $\forall$]
5. From 4., for some $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_1 , then ' $\exists y \neg G(\mathfrak{m}, y)$ ' is false on I_1 . [Def Sat.]
6. From 5., for some $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_1 , every $\beta \in D$ does not satisfy ' $\neg G(\mathfrak{m}, y)$ ' on I_1 . [TC $\exists$]
7. From 6., for some $\alpha \in D$, every $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_1 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $\neg G(\mathfrak{m}, \mathfrak{n})$ ' is false on I_1 . [Def. Sat.]
8. From 7., for some $\alpha \in D$, every $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_1 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $G(\mathfrak{m}, \mathfrak{n})$ ' is true on I_1 . [TC $\neg$]
9. From 8., for some $\alpha \in D$, every $\beta \in D$ is such that $\langle \alpha, \beta \rangle \in "G"$ on I_1 . [TC Atomic]
10. From 2., some $\gamma \in D$ does not satisfy " $\exists y G(x, y)$ " on I_1 . [TC $\forall$]
11. From 10., for some $\gamma \in D$, if ' $\mathfrak{m}$ ' designates γ on I_1 , then ' $\exists y G(\mathfrak{m}, y)$ ' is false on I_1 . [Def. Sat.]
12. From 11., for some $\gamma \in D$, if ' $\mathfrak{m}$ ' designates γ on I_1 , every $\beta \in D$ does not satisfy ' $G(\mathfrak{m}, y)$ ' on I_1 . [TC $\exists$]

13. From 12., for some $\gamma \in D$, every $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates γ on I_1 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $G(\mathfrak{m}, \mathfrak{n})$ ' is false on I_1 . [Def. Sat.]
14. From 13., for some $\gamma \in D$, every $\beta \in D$ is such that $\langle \gamma, \beta \rangle \notin "G"$ on I_1 . [TC Atomic]
 [Note: Both of the conditions on I_1 (in lines 9. and 14.) are being "driven" by a restricted specification of an object. That means the ordered pairs indicated by each condition must start with the same object (since the condition is not just there being some object or other for each object in D)—separately for each condition, but not across the two conditions. These "common" objects must each get paired with every object from the domain and (for 9.) be included in the extension of " G " or (for 14.) be excluded from the extension of " G ". This tells us that a domain of one object will not work here, since that would require both that $\langle 1, 1 \rangle \in "G"$ on I_1 and that $\langle 1, 1 \rangle \notin "G"$ on I_1 . So D must have at least two objects.]
- By 9. and 14., have I_1 as follows: $D = \{1, 2\}$, " G ": $\{\langle 1, 1 \rangle, \langle 1, 2 \rangle\}$. [This meets the condition on I_1 in 9., by having 1 paired with each object in D . Moreover, since it automatically leaves $\langle 2, 1 \rangle$ and $\langle 2, 2 \rangle$ out of the extension of " G ", it also meets the condition on I_1 in line 14. by leaving out 2 paired with each object in D .] This makes the first sentence true and the second sentence false, so they are semantically inequivalent. [No need to consider Part II.]

BONUS EXAMPLE: I actually meant Additional Practice Problem #6 to be different, but I typed it in wrong. So, here is the very similar problem I meant to give you.

7. Is $\neg \forall y \exists x \neg G(x, y) \models \forall x \exists y G(x, y)$? (That is, are the two sentences equivalent or inequivalent?)
 [Note the difference in how the variables are attached to the quantifiers on the lhs.]

Check for inequivalence, that is, check whether there is some I_1 that makes the first true and the second false, or some I_2 that makes the first false and the second true.

Part I: I_1 makes

1. " $\neg \forall y \exists x \neg G(x, y)$ " true AND
2. " $\forall x \exists y G(x, y)$ " false.
3. From 1., I_1 makes " $\forall y \exists x \neg G(x, y)$ " false. [TC $\neg$]
4. From 3., some $\alpha \in D$ does not satisfy " $\exists x \neg G(x, y)$ " on I_1 . [TC $\forall$]
5. From 4., for some $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_1 , then ' $\exists x \neg G(x, \mathfrak{m})$ ' is false on I_1 . [Def. Sat.]
6. From 5., for some $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_1 , every $\beta \in D$ does not satisfy ' $\neg G(x, \mathfrak{m})$ ' on I_1 . [TC $\exists$]
7. From 6., for some $\alpha \in D$, every $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_1 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $\neg G(\mathfrak{n}, \mathfrak{m})$ ' is false on I_1 . [Def. Sat.]
8. From 7., for some $\alpha \in D$, every $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_1 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $G(\mathfrak{n}, \mathfrak{m})$ ' is true on I_1 . [TC $\neg$]
9. From 8., for some $\alpha \in D$, every $\beta \in D$ is such that $\langle \beta, \alpha \rangle \in "G"$ on I_1 . [TC Atomic]
10. From 2., some $\gamma \in D$ does not satisfy " $\exists y G(x, y)$ " on I_1 . [TC $\forall$]
11. From 10., for some $\gamma \in D$, if ' $\mathfrak{m}$ ' designates γ on I_1 , then ' $\exists y G(\mathfrak{m}, y)$ ' is false on I_1 . [Def. Sat.]
12. From 11., for some $\gamma \in D$, if ' $\mathfrak{m}$ ' designates γ on I_1 , every $\beta \in D$ does not satisfy ' $G(\mathfrak{m}, y)$ ' on I_1 . [TC $\exists$]

13. From 12., for some $\gamma \in D$, every $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates γ on I_1 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $G(\mathfrak{m}, \mathfrak{n})$ ' is false on I_1 . [Def. Sat.]

14. From 13., for some $\gamma \in D$, every $\beta \in D$ is such that $\langle \gamma, \beta \rangle \notin "G"$ on I_1 . [TC Atomic]

[Note: Both of the conditions on I_1 (in lines 9. and 14.) are (again) being "driven" by a restricted specification of an object. Thus, this object must, for each condition separately, be the same object for the pairs that each condition indicates (since the condition is not just there being some object or other for each object in D). In each condition, this "common" object gets paired with every object in the domain. But in 9., the "common" object is in the *second* position in the pairs, while in 14. it is in the *first* position. So, 9. tells us to include every pair that ends with a certain object, while 14. tells us to exclude every pair that begins with a certain object. But this inclusion demand and exclusion demand will overlap, not just for a domain of only one object, but for any domain.]

Both α and γ are restricted, and the conditions on I_1 are relativized to them. β is unrestricted, so consider first when β is α and then when β is γ . When β is α , then, by 9., $\langle \alpha, \alpha \rangle \in "G"$ on I_1 , and, by 14., $\langle \gamma, \alpha \rangle \notin "G"$ on I_1 . When β is γ , then, by 9., $\langle \gamma, \alpha \rangle \in "G"$ on I_1 , and by 14., $\langle \gamma, \gamma \rangle \notin "G"$ on I_1 . Since $\langle \alpha, \alpha \rangle \in "G"$ on I_1 , but $\langle \gamma, \gamma \rangle \notin "G"$ on I_1 , α and γ must be distinct objects. But even if I_1 involves two (or more) objects, it still has $\langle \gamma, \alpha \rangle \notin "G"$ on I_1 and $\langle \gamma, \alpha \rangle \in "G"$ on I_1 , where α and γ are each the same in both cases (they are "driving" the conditions). That's a contradiction, so no I makes the first sentence true and the second sentence false. [Recalling that logical equivalence is mutual entailment, this means that there is entailment from left-to-right. So now we have to consider whether there is entailment from right-to-left.]

Part II: I_2 makes

1. " $\neg \forall y \exists x \neg G(x, y)$ " false AND
2. " $\forall x \exists y G(x, y)$ " true.
3. From 1., I_2 makes " $\forall y \exists x \neg G(x, y)$ " true. [TC $\neg$]
4. From 3., every $\alpha \in D$ satisfies " $\exists x \neg G(x, y)$ " on I_2 . [TC $\forall$]
5. From 4., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_2 , then ' $\exists x \neg G(x, \mathfrak{m})$ ' is true on I_2 . [Def. Sat.]
6. From 5., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_2 , some $\beta \in D$ satisfies ' $\neg G(x, \mathfrak{m})$ ' on I_2 . [TC $\exists$]
7. From 6., for every $\alpha \in D$, some $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_2 and ' $\mathfrak{n}$ ' designates β on I_2 , then ' $\neg G(\mathfrak{n}, \mathfrak{m})$ ' is true on I_2 . [Def. Sat.]
8. From 7., for every $\alpha \in D$, some $\beta \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_2 and ' $\mathfrak{n}$ ' designates β on I_1 , then ' $G(\mathfrak{n}, \mathfrak{m})$ ' is false on I_2 . [TC $\neg$]
9. From 8., for every $\alpha \in D$, some $\beta \in D$ is such that $\langle \beta, \alpha \rangle \notin "G"$ on I_2 . [TC Atomic]
10. From 2., every $\alpha \in D$ satisfies " $\exists y G(x, y)$ " on I_1 . [TC $\forall$]
11. From 10., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_2 , then ' $\exists y G(\mathfrak{m}, y)$ ' is true on I_2 . [Def. Sat.]
12. From 11., for every $\alpha \in D$, if ' $\mathfrak{m}$ ' designates α on I_2 , some $\gamma \in D$ satisfies ' $G(\mathfrak{m}, y)$ ' on I_2 . [TC $\exists$]
13. From 12., for every $\alpha \in D$, some $\gamma \in D$ is such that, if ' $\mathfrak{m}$ ' designates α on I_2 and ' $\mathfrak{n}$ ' designates γ on I_2 , then ' $G(\mathfrak{m}, \mathfrak{n})$ ' is true on I_2 . [Def. Sat.]

14. From 13., for every $\alpha \in D$, some $\gamma \in D$ is such that $\langle \alpha, \gamma \rangle \in \text{"G"}$ on I_2 . [TC Atomic]
 [The conditions on I_2 are being driven by unrestricted specifications, making the restricted specifications they include relativized to that unrestricted "ranging". That means that every object in the domain must get paired with *some object or other*—but not necessarily with the same object as the one any other object from D gets paired with.]

α is unrestricted, while β and γ are restricted but (separately) relativized to α . So, line 9. requires that we exclude from the extension of "G" a pair that ends with each object in D , starting with some object or other in each case, but not necessarily the same object. Line 14. requires that we include a pair that starts with every object in D , paired with some object or other but not necessarily the same object. This again means a domain of only one object is insufficient, but it works if we have two (or more) objects. Set I_2 as follows: $D = \{1, 2\}$, "G": $\{\langle 1, 2 \rangle, \langle 2, 1 \rangle\}$. [This directly satisfies 14. by including a pair starting with each object paired with some object or other. By leaving out $\langle 1, 1 \rangle$ and $\langle 2, 2 \rangle$, it also satisfies 9., since it excludes a pair ending with each object in D , paired with some object or other.] Since this I_2 makes the first sentence false and the second sentence true, it shows they are inequivalent.