What was Frege Trying to Prove? A Response to Jeshion

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Frege set out to identify the foundations of arithmetic. The project he envisioned required the construction of gapless proofs of the basic truths of arithmetic from laws of logic and definitions. But why did he look for the foundations of arithmetic in *logic*? Why was he not content to take, as foundational axioms of arithmetic, its basic truths—for example, that every natural number has a unique successor?

In the 1970s and 1980s, Paul Benacerraf, Philip Kitcher, and I offered accounts of Frege's motivation that were designed to answer these questions.[1] We all took Frege at his word when he said, in section 3 of *Foundations*, that his inquiry was prompted by his desire to determine whether the truths of arithmetic are analytic or synthetic, a priori or a posteriori. But we offered three competing interpretations of these distinctions. Kitcher and I both argued that they had important ties to a philosophical tradition.[2] Benacerraf, in contrast, argued that these apparently philosophical distinctions, as reinterpreted by Frege, belonged to a mathematical rather than a philosophical tradition.

In 'Frege's Notions of Self Evidence', Robin Jeshion argues that all three of us have gone wrong. The problem, according to Jeshion, is that we have either ignored or misinterpreted Frege's appeals to self-evidence. A correct account of his motivation, she argues, is to be found by looking at these appeals. This account requires no reference to the analytic/synthetic and a priori/a posteriori distinctions. For these 'specifically philosophical notions of section 3', she claims, do not often appear later in his writings (Jeshion 2001, p. 938).[3]

[1] See Benacerraf 1981; Kitcher 1979, 1986; Weiner 1984, 1990. Although the account I gave in Weiner 1990 is, in many respects a development of the 1984 account, my views changed in the interim. Thus there are also significant differences.

[2] For my objections to Kitcher's interpretation, see Weiner 1990, pp. 55–63.

[3] Jeshion's interpretation follows a recent trend in Frege interpretation, exemplified in the writings of Mark Wilson and Jamie Tappenden (See, for example, Tappenden 1995a, 1995b; Wilson 1992).

Mind, Vol. 113 . 449 . January 2004

I shall argue that this is a mistake. The 'Euclidean Rationale' that Jeshion attributes to Frege not only contradicts many of his actual claims, it also fails to answer the questions about Frege's commitment to logicism. The problem, I believe, lies with Jeshion's desire to de-emphasize the importance of the analytic/synthetic and a priori/a posteriori distinctions. These distinctions play a central role in Frege's understanding of his project—a role that is not played by Frege's appeals to self-evidence. Any account of what motivates Frege's search for logical foundations for arithmetic that ignores these distinctions will fall short. I shall also respond to Jeshion's criticisms of my own interpretation.

Although there are many respects in which my interpretation and Jeshion's are at odds, it may be useful to begin with some respects in which we (and virtually all Frege's readers) agree. Frege wants to define all terms of arithmetic from primitive, undefinable terms. He wants to construct a list of axioms or primitive truths from which all truths of arithmetic can be proved by gapless logical inferences. And he wants to prove some of the basic truths of arithmetic from primitive truths. To do this is to provide, in Frege's later terminology, a systematic science of arithmetic. Science, he claims, comes to fruition only in a system (Frege 1914, NS p. 261/PW p. 242). But this metaphorical remark does not really tell us why Frege wants a systematic science of arithmetic.

Jeshion describes, as follows, the 'Euclidean Rationale' that, she claims, motivates Frege's project:

> **Euclidean Rationale:** Frege thought the primitive truths of mathematics have two properties. (i) they are *selbstverstandlich*; foundationally secure, yet are not grounded on any other truth, and, as such, do not stand in need of proof. (ii) And they are self-evident; clearly grasping them is a sufficient and compelling basis for recognizing their truth. He also thought that the relations of epistemic justification in a science mirror the natural ordering of truth: in particular, what is self-evident is *selbstverstandlich*. Finding many propositions of arithmetic non-self-evident, Frege concluded that they stand in need of proof. (Jeshion 2001, pp. 944, 961–2)

Tappenden and Wilson argue that Frege's motivation cannot be understood except against a background of the history of geometry. These writers do not deny that Frege's motivation was, in part, philosophical. Tappenden, in particular, is adamant that it was, but the suggestion in his approach is that there is no philosophical strand in this motivation that is not *also* part of a mathematical tradition. In offering an explanation of Frege's motivation that makes no reference to the analytic/synthetic and a priori/a posteriori distinctions, Jeshion takes this idea even further.

No mention is made of the analytic/synthetic and a priori/a posteriori distinctions and, on Jeshion's interpretation, these distinctions play no important role in Frege's conception of the project. Frege's systematic science, on Jeshion's interpretation, is needed to exhibit the natural ordering of truths of mathematics. But this natural ordering of truths might well stop with the basic truths of arithmetic rather than recognizable laws of logic. Why then think the basic truths of arithmetic should be proved from laws of logic? Jeshion's answer to this question is that primitive truths must be self-evident and Frege found many propositions of arithmetic non-self-evident.

In what follows I will focus on two central features of Jeshion's interpretation. One is that the import of systematization, for Frege, is in its exhibition of the natural ordering of truths. The other is that many propositions of arithmetic are not self-evident.

Let us begin with the second of these. As stated, of course, it is much too weak to motivate Frege's project. There are many truths of arithmetic that are not self-evident—otherwise, number theory would not be a field of mathematics. If lack of self evidence is to play an important role in motivating Frege's project, it must not simply be that many truths of arithmetic are not self-evident but, rather, that the basic truths of arithmetic—the truths that might be taken as axioms—are not self-evident. What support does Jeshion offer for this part of her interpretation?

Jeshion claims that Frege offers 'arguments for thinking the simplest propositions of arithmetic are not self-evident' (Jeshion 2001, p. 963). But in the sections to which she alludes, those immediately following section 4, there are no such arguments. There is no discussion of the self-evidence of the simplest truths of arithmetic. Frege there criticizes Kant's claim that truths of arithmetic are synthetic a priori. Kant, Frege says, takes the numerical formulae as unprovable. Frege claims that it is not self-evident that $135664 + 37863 = 173527$, but that this is not a reason to believe, as Kant does, that the truth is synthetic a priori. It is, rather, a reason to believe that the truth is provable. This formula, however, is hardly the sort of truth that might be taken as an axiom of arithmetic. Does Frege offer other arguments about the non-self-evidence of such truths?

It is clear from the discussions of sections 6 and 7 of *Foundations* what he thinks these truths are. All numbers, he says, are definable from one and increase-by-one. He then suggests that all numerical formulae will be derivable from general laws about numbers, in addition to definitions of the numbers from one and increase-by-one. The general laws include laws of arithmetic, for example, $a+(b+1) = (a+b)+1$.

We can see from this that the basic truths of arithmetic will include general laws of arithmetic as well as facts about the number one and increase-by-one. But is there evidence that Frege believes that *these truths* are not self-evident? If there is, Jeshion has offered us none. In the sections of *Foundations* immediately following section 4—those in which, Jeshion claims, Frege argues that the simplest truths of arithmetic are not self-evident—he argues no such thing. It is crucial to Jeshion's account that Frege thinks the basic truths of arithmetic are not self-evident. Yet she provides no textual evidence that he *did* think this.

One might think, however, that this is no criticism of Jeshion's interpretation—that it is a mistake to expect such evidence. After all, Frege apparently never considers the possibility of constructing a systematic science whose axioms are familiar truths of arithmetic. Perhaps it is simply an underlying assumption that the basic truths of arithmetic are *not* self-evident.

But such an interpretation does not fit Frege's texts. Frege does think that what is not self-evident should be proved. But he does not take it as evident, in *Foundations*, that the basic propositions of arithmetic require proof. He writes, not that the fundamental propositions of arithmetic [*die Grundsätze der Arithmetik*] should be given rigorous proofs but, rather, that they should be proved, *if in any way possible* [*wenn irgend möglich*], with the utmost rigour; (Frege 1884, p. 4 emphasis added). As he indicates, in the next two paragraphs, the discussions that immediately follow are designed to address issues that will give 'a pointer' to answering the questions 'Is the concept of number definable?' and 'Are the basic propositions of arithmetic provable?' Frege *does* explicitly discuss the issue of whether or not the basic propositions of arithmetic are provable. One of his aims is to convince the reader that these propositions are provable. If Frege's reason for thinking that these propositions are provable is that they are not self-evident, we would expect this consideration to come up in his discussions of the issue. It does not.

Granted, as Jeshion has shown, Frege does say that certain propositions, because they are not self-evident, should be proved. The problem, however, is that these are not basic propositions of arithmetic. Moreover, there is textual evidence about Frege's view—evidence that goes against Jeshion's interpretation. For Frege suggests, in section 14 of *Foundations*, that if we try to deny one of the fundamental propositions of arithmetic [*Grundsätzen der Zahlenwissenschaft*], 'even to think is no longer possible'. And he does not mean thereby to be making a psychological claim.[4,5]

Why, then, should Frege's systematic science not stop with the fundamental propositions of arithmetic? He is, as I have indicated above, not silent on the issue. Gapless proofs of a truth from primitive truths will, he claims, tell us whether the truth in question is analytic or synthetic, a priori or a posteriori (Frege 1884, pp. 3–4). And he writes:

> [S]uppose there should prove to be grounds from other points of view for believing that the fundamental principles of arithmetic are analytic, then these would tell also in favour of their being provable … while any grounds for believing the same truths to be a posteriori would tell in the opposite direction. (Frege 1884, p. 5)

Indeed, Frege's discussion of whether numerical formulae are self-evident in section 5, the discussion to which Jeshion appeals, appears in the context of a discussion of Kant's claim that these truths are synthetic a priori. Frege's claim in section 3 of *Foundations*, that his investigations were prompted by a desire to determine whether the truths of arithmetic are analytic or synthetic, a priori or a posteriori is not mere window dressing. The issue of how the truths of arithmetic are to be classified figures, also, in his characterization of what he hopes to have accomplished in *Foundations*. This is, he says, 'to have made it probable that the laws of arithmetic are analytic judgments and consequently a priori' (Frege 1884, p. 99). Only actual gapless proofs from primitive logical truths, however, will show that the laws of arithmetic are analytic. These proofs are to be provided in a later work, *Basic Laws of Arithmetic*. That is,

[4] Frege is here discussing what can be denied in conceptual thought. He contrasts the basic laws of arithmetic with axioms of geometry. We can, he says, assume the contrary of an axiom of geometry 'without involving ourselves in any self-contradictions when we proceed to our deductions' (Frege 1884, p. 20). This is not a claim about our psychology but, rather, about correct deductions. He goes on to consider the possibility of trying to deny a basic law of arithmetic and asks 'would thought even be possible? [*Wäre dann noch Denken möglich*]'. He then concludes 'the basis of arithmetic lies deeper, it seems, than that of any of the empirical sciences, and even than that of geometry' and suggests that the laws of numbers are 'connected very intimately with the laws of thought [*Gesetze … des Denkens*] (Frege 1884, p. 21). This use of 'laws of thought' is entirely consonant with Frege's use of the expression throughout his writings. And, when the topic comes up later, he is adamant both that the laws of thought are not psychological and that we can learn nothing about laws of thought from studying our psychological characteristics (See, for example, Frege 1893, pp. xv–xvii). Thus Frege's remark does not seem to be about our psychology.

[5] I have been using the term 'self-evident' in an everyday non-psychological sense, rather than in either of the technical senses that Jeshion introduces and attributes to Frege. I have not, in particular, argued that the Peano axiom is self-evident in either the (S-E) or (S-V) sense. But my practice, in using this non-technical term is no different from Frege's. As Jeshion acknowledges, Frege's own use of the terms translated by 'self-evident' are 'non-technical' (Jeshion 2001, p. 938). The matter of whether he in fact is committed to the two technical notions Jeshion describes turns on the cogency of her interpretation.

Frege's systematic science of arithmetic is supposed to tell us whether the truths of arithmetic are analytic or synthetic, a priori or a posteriori. And, viewed in this way, we can see why our familiar propositions of arithmetic should not, on Frege's view, be taken as primitive truths of this systematic science.

To see this, it will help to note that another way to describe Frege's understanding of this system of classification is to describe it as one that categorizes propositions according to the sort of ultimate ground that must be invoked in order to justify them.[6] All justification requires thought. The mark of an analytic truth is that it requires *only* thought. Some justification requires, in addition, appeal to inner intuition; and some also requires appeals to sense perception. For Frege, these distinctions mark out a hierarchy of generality.[7] The analytic truths are the most general truths (those that govern everything), and the synthetic a posteriori, the least general.

How are we to categorize the truths of arithmetic? The status of a truth depends on what it takes to construct a gapless proof of the truth from primitive truths (Frege 1884, p. 4). If such a proof requires only general logical laws and definitions, the truth is analytic. If the proof requires truths that 'belong to the sphere of some special science', it is synthetic. And if the proof requires an appeal to facts (assertions about particular objects), it is a posteriori.

So far, the views I have attributed to Frege appear explicitly in his writings. But there is another issue that Frege does not explicitly address. Suppose, for the moment, we have a systematic science of arithmetic. Do we now know whether its truths are analytic or synthetic, a priori or a posteriori? Or is further work required? The answer will turn on what is required to identify the category of the primitive truths on which its proofs are based. Assuming these truths do not require proof, there are two possibilities. Either the category of a primi-

[6] As I wrote in Weiner 1990 p. 51. Although it is true that Frege rarely mentions the analytic/synthetic and a priori/a posteriori distinctions in his later writings, this is merely symptomatic of a change in vocabulary. Talk about these distinctions is replaced by talk about sources of knowledge, but the concerns are exactly the same.

It is worth noting that Jeshion does not object to the claim that a concern with sources of knowledge is involved in his motivation. Indeed she refers to my account of Frege's motivation as the Knowledge-of-Sources Rationale and she writes 'I can (and do) comfortably attribute to him both Mathematical Rationale and the Knowledge-of-Sources Rationale' (Jeshion 2001, p. 944). The real difference here between Jeshion's view and mine seems to be that Jeshion thinks the concern with sources of knowledge cannot, on its own, explain why Frege wants proofs of the truths of arithmetic from acknowledged logical laws. The Knowledge-of-Sources Rationale, on Jeshion's view, is not a part of the Euclidean Rationale it merely points to another (less important) concern of Frege's.

[7] As I argue in Weiner 1990 p. 51. For textual support see, for example, Frege 1884, sections 3 and 14.

tive truth is evident from its content alone (that is, no further work is required) or there is some other procedure for identifying its category (further work is required). Given that Frege does not address this issue, what can we say about his view?

Frege's *Foundations* description of how we determine the status of a truth suggests that no further work is required. He never suggests that, after finding a proof from primitive truths, we must engage in an investigation to determine whether these truths are general logical laws, truths of a special science or assertions about particular objects. Nor does he engage in such an investigation in his attempt to show us that the truths of arithmetic are analytic. Moreover, he certainly seems to think that there are truths whose category is evident from their content alone. For these reasons, I claimed in *Frege in Perspective* that, on Frege's view, it is a hallmark of primitive truths that the category to which they belong should be evident from their content.[8]

Let us now return to the question with which we began—why not take it as a foundational axiom of arithmetic that every natural number has a unique successor? Suppose we have gapless logical proofs of truths of arithmetic from this axiom. Can we tell whether these truths are analytic or synthetic, a priori or a posteriori? It depends on the status of our axiom. And a brief reflection shows that its status is not obvious. On the one hand, it does not seem to belong to a particular special science. After all, truths of arithmetic are employed in virtually all special sciences. But on the other hand, it does not seem to have the requisite generality. Instead of governing the unrestricted domain, as Frege claims analytic truths do (Frege 1884, p. 21), it seems to govern only the peculiar domain of the numbers. Thus the status of the claim that every natural number has a unique successor is not evident from its content.

Nonetheless, Frege was convinced that the truths of arithmetic really are analytic. He writes:

> We therefore have no choice but to acknowledge the purely logical nature of arithmetical mode of inference. Together with this admission, there arises the task of bringing this nature to light wherever it cannot be recognized immediately … (Frege 1885, pp. 95–6)

To show that the truths of arithmetic really are analytic, we need gapless proofs from primitive logical laws—logical laws whose status *as* logical laws is evident from their content.

[8] This view is the focus of Jeshion's central criticism of my interpretation. She claims, correctly, that I offer no textual support for this view in *Frege in Perspective*. Although the reasons I advance above were my reasons for adopting this view in *Frege in Perspective*, I did not actually argue for the view there.

It is important to note that, although Frege was entirely convinced that the truths of arithmetic are analytic, he was also entirely serious in his view that such proofs are necessary to show that the truths of arithmetic are analytic. Indeed, in his last years he took the failure of his original project as evidence that the truths of arithmetic are not really analytic but, rather, synthetic a priori (see, for example, Frege 1924/1925).

Let us turn next to Jeshion's characterization of Fregean systematization. I have argued that Frege's demand for a systematic science arises from a desire to identify sources of knowledge. Systematization, however, need not be understood in this way. It is a recognizable mathematical project—one that can be, and often is, undertaken without any peculiarly philosophical motivation. And Jeshion seems to think that this is how Frege's demand for a systematic science should be understood. But, again, her account does not fit with Frege's writings.

Jeshion agrees that, on Frege's view, even empirical sciences should be systematized. The basis of a systematic science is its primitive truths. The primitive truths of a systematic science, Jeshion claims, are not only *selbstverstandlich* [foundationally secure, yet not grounded on any other truth; not in need of proof], in addition

> they contain constituents that denote the simples of their respective disciplines, they possess a universality and basicness appropriate to their discipline, and their truth is not dependent on any other, more basic, truth of that discipline. (Jeshion 2001, p. 948)

Jeshion says, without offering textual support, that in a systematic science the more specific truths are derived from the more general.[9] She claims, citing *Foundations* section 3, that Frege says primitive a posteriori truths 'cannot be proved' (Jeshion 2001, p. 948), and adds that the reason 'seems simply to be that they are the most general and basic laws of their discipline'.[10] But this does not fit with what little Frege says about empirical science.

First, the claim about the passage from section 3 is simply wrong. Frege writes:

[9] Jeshion 2001, p. 945. Her only citation is to 'Logical Generality' PW 258. Frege does say there that the value a law has for our knowledge 'rests on the fact that it comprises many—indeed, infinitely many—particular facts as special cases. We profit from our knowledge of a law by gathering from it a wealth of particular pieces of information, using the inference from the general to the particular'. But there is no talk of systematic science or the natural order of truths.

[10] Here she cites 'Logical Generality', p. 258. But although Frege does say there that the value a law has rests in its comprising many particular facts, he does not talk about systematic science or about primitive truths. Nor does he mention what justifies the general laws.

> For a truth to be a posteriori, it must be impossible to construct a proof of it without including an appeal to facts, that is, *to truths which cannot be proved and are not general, since they contain assertions about particular objects.* (Frege, 1884, p. 4 emphasis added)

If it is impossible to construct a proof of an a posteriori truth without appeal to facts, then at least some of these facts are, presumably, primitive truths. According to Jeshion primitive truths are always general laws. But are these facts general laws? Frege says that, since they contain assertions about particular objects, they are not general. And, as he makes clear when he argues that the laws of arithmetic are not inductive truths, the justification of a posteriori truths requires particular observations of particular objects (Frege 1884, sects 9–10). Whatever we may think of this odd talk of proving a posteriori truths, it is evident that Frege thinks the justification of a posteriori truths—including laws of natural sciences—requires appeals to data, particular facts about particular objects. Moreover, he thinks this appeal needs to be revealed by a systematization of that science. Thus the primitive truths of a systematic natural science will, it seems, include supporting data for its laws.[11]

What this shows is that the concern with justification in a science is distinct from the concern with what Jeshion identifies as natural ordering: 'a structuring of propositions in part according to their relative simplicity and complexity, and their generality and specificity' (Jeshion 2001, p. 945). A systematic natural science that gives us the best possible justification of its laws will be different from a systematic science in which specific truths can only be derived from general laws. If Frege's interest is in the former—and there is every indication that it is—neither universality nor *selbstverstandlichkeit* in Jeshion's sense is a general characteristic of primitive truth.

What about mathematics? Since no appeals to facts are required, one might suppose that here justification and natural order (in Jeshion's sense) coincide. Jeshion claims that Frege thinks primitive truths of mathematics must be maximally general (Jeshion 2001, p. 950). Jeshion is not explicit about what she means by 'maximally general'. A maximally general law, as this expression is usually used, is one that applies everywhere rather than to a limited domain. For Frege, *all* truths of

[11] This is less odd than it sounds, if we remember that Frege recognized the possibility of alternate systems for any science. Thus different systems for a natural science may have different data as their primitive truths. Each system, however, will provide adequate grounds for the general laws.

arithmetic are maximally general in this sense.[12] But this generality cannot be maximal on Jeshion's 'natural order' interpretation of systematization. For, on this interpretation, some truths of arithmetic must be less general than others. What evidence is there that Frege held such a view?

Jeshion gives one example. She claims that Frege thinks propositions formed from stipulative definitions cannot be axioms because they lack universality. But some statements formed from stipulative definitions *are* general statements with unrestricted domain.[13] Why does Jeshion say that Frege thinks they lack universality? Her evidence comes from a passage in which Frege claims that stipulative definitions cannot be counted among the principles of a discipline. He says, a definition's 'epistemic value [*Erkenntniswert*] is no greater than that of an example of the law of identity $a=a$' (Frege 1903, p. 320). But the fact that particular propositions of the form '$a=a$' lack epistemic value has nothing to do with their lacking universality.[14] For, in the opening paragraph of 'On Sense and Meaning', Frege famously distinguishes those propositions from propositions of the form '$a=b$'—the former lack *Erkenntniswert*, while the latter can contain valuable extensions of our knowledge. But propositions of the form '$a=b$' are no more universal than those of the form '$a=a$'.

Jeshion provides no evidence that Frege has the fine-grained notion of generality he must have if the 'natural order' interpretation of Fregean systematization is to work. Perhaps it is meant to be something trivial—for example, if A is provable from B and B is not provable from A, then B precedes A in the natural order. But this is not really a notion of generality. Moreover, were Frege's aim to introduce a system that expresses a natural order in this sense, he would surely have listed $A \rightarrow A$ as a theorem, deriving it from Basic Law I: $A \rightarrow (B \rightarrow A)$. Instead,

[12] Or, at least, this is what he thinks in the pre-contradiction years. It is important to note that this is part of what will be shown by Frege's definitions. Once the numerals and the concept number are defined using primitive terms of Frege's logic, statements about numbers can be expressed in a way that makes it clear that they have unrestricted domain.

[13] To see this, consider a proposition formed from one of Frege's first definitions: the definition of the Begriffsschrift sign for which Frege used the term 'equinumerate' [gleichzahlig]. A natural language statement of the definition, as it appears in Frege's proofs would be:

> The concept *F* is equinumerate with the concept *G* if and only if there exists a relation *F* that correlates one to one the objects falling under the concept *F* with the objects falling under the concept *G*

This statement has unrestricted domain.

[14] It is true that Frege contrasts a particular instance of this general law of identity with the law itself. But he also expresses hesitation about calling the general law an axiom—evidently for the same reason: that it cannot add to our knowledge.

he includes both on his list of basic laws. It is not difficult to see why. Although Frege wants to minimize the number of basic laws, he also wants to use them to construct proofs. He is faced with the usual trade-off between an easily surveyable set of axioms and ease of proof within the system. This is one of the reasons Frege says that there is no unique correct system. A theorem in one system may be an axiom in another.

Jeshion acknowledges this and says,

> So, in saying that primitive truths can be axioms, I mean that for each primitive truth, there is a systematization that takes it as one of its axioms. (Jeshion 2001, p. 951)

But if this is so, Jeshion has offered us no answer to our original question. Why should Frege not include the Peano axiom with which we began? Why can there not be a system whose axioms include Frege's basic laws and the Peano axioms? Jeshion may seem to have an answer. She writes,

> The simples constitute the essence of a discipline. That geometry is, according to Frege, essentially spatial entails that its ultimate building blocks are spatial configurations. That arithmetic is essentially general, governing (applying to) everything, entails that its ultimate building blocks are purely logical. (Jeshion 2001, p. 947)

If arithmetic is maximally general then, it may seem, a systematic science of arithmetic will have as axioms only logical laws. Hence, the basic truths of arithmetic must be proved from logical laws. But the problem with this answer is that we are still missing an explanation of what is wrong with the Peano axioms. Supposing (as, we agree, he initially does) that arithmetic *is* maximally general, then such concepts as 'number' *are* logical concepts; laws of number (including the Peano axioms) *are* logical laws.

The problem with a system based on the Peano axiomatization is not that the axioms are not logical laws. Such basic arithmetical truths as the Peano axioms require proof because, as I indicated above, it is not *evident* that the Peano axioms are logical laws. One might suspect that they govern a restricted realm—the realm of natural numbers. Proving Peano axioms from recognizably logical laws is part of the 'task of bringing [the purely logical] nature [of arithmetical mode of inference] to light wherever it cannot be recognized immediately' (Frege 1885, pp. 95–6). In order to do this, Frege needs to derive the basic truths of arithmetic from truths whose status is evident from their content: primitive truths.

This view about primitive truths is the feature of my interpretation that Jeshion finds most objectionable. There is, she thinks, no textual evidence for it. But, as I have tried to show, we cannot tell a satisfying story about why Frege wants to prove the basic truths of arithmetic from acknowledged logical laws *unless* we attribute some such view to him. Or, at least, if we can, Jeshion has not done so.

Jeshion's interpretation appears to take it inspiration from the opening sentence of section 1 of *Foundations*. Frege writes:

> After deserting for a time the old Euclidean standards of rigor, mathematics is now returning to them, and even making efforts to go beyond them.

And he writes, later in the paragraph, 'Proof is now demanded of many things that formerly passed as self-evident'. There can be no question that this is an important part of Frege's motivation. But the call to return to Euclidean standards is not a general demand that *everything* be proved. We need to know how to recognize when proofs should stop. We need to know, in particular, why we should not stop with familiar basic laws of arithmetic. And, here, Jeshion has not provided us with a convincing answer. She says primitive truths must be self-evident but offers no evidence that Frege does think basic truths of arithmetic are not self-evident.[15] She says primitive truths must be maximally general but offers no evidence that he thinks basic truths of arithmetic are not maximally general.

What prevents Jeshion from coming up with an answer to her central question is, I suspect, her desire to avoid attributing to Frege any views that carry with them the taint of the purely philosophical.[16] Instead, she focuses on a notion that will be familiar and recognizable to any mathematician: self-evidence. But she would have done well to heed Frege's comments about mathematicians and self-evidence. He writes:

> … the mathematician rests content if every transition to a fresh judgement is self-evidently correct (*richtig einleuchtet*), without enquiring into the nature of this self-evidence (*dieses Einleuchtens*), whether it is logical or intuitive. (Frege 1884, pp. 102–3) [17]

[15] It may be worth noting that I do not mean to suggest that Frege thinks primitive truths of mathematics need not be self-evident. I agree with Jeshion that Frege thinks primitive truths of mathematics must be self-evident. My objection is to Jeshion's claim that Frege thought the basic truths of arithmetic were not self-evident.

[16] Hence her preference for talk about 'sources of knowledge' (which she takes to be both philosophical and mathematical) over talk of the analytic/synthetic and a priori/a posteriori distinctions.

[17] See also, Frege 1893, p. 29 and the 1902 letter to Huntington.

Frege was after self-evidence, but he was also after something more. He writes, in the introduction to *Basic Laws*:

> Because there are no gaps in the chains of inference, every 'axiom', every 'assumption', 'hypothesis', or whatever you wish to call it, upon which a proof is based is brought to light; and in this way we gain a basis upon which to judge the epistemological nature of the law that is proved (*der Erkenntnisstheoretischen Natur des bewiesenen Gesetzes*). (Frege 1893, p. vii)

And, as this indicates, he was after answers to distinctly philosophical questions—that is. to use Benacerraf's words:

> [E]pistemological and metaphysical questions that arise in accounting for [a] body of knowledge, fitting it into a general account of knowledge and the world. (Benacerraf, 1981, p. 23)

I do not mean to suggest that no mathematician does or can seek answers to these questions. On the contrary, Frege was just such a mathematician—and one who hoped to engage other mathematicians in his philosophical quest. What I have been arguing here is that the fact that Frege is asking distinctly philosophical questions must be recognized if we are to make sense of what he wrote. What I have not argued for here, but believe no less strongly, is that what we get from this act of recognition is not just a more faithful reading of Frege's text; we also gain access to an exceedingly deep and important contribution to philosophy.[18]

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[18] I am indebted to Mark Kaplan for extensive discussions and comments on earlier drafts. I would also like to thank an anonymous referee for many helpful suggestions. This paper was written during a period of funding by the Guggenheim Foundation. I am very grateful for their support.

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